PMSM system description

March 24, 2011

1 Model of PMSM Drive

Permanent magnet synchronous machine (PMSM) drive with surface magnets on the rotor is described by conventional equations of PMSM in the stationary reference frame:

$$
\begin{aligned}
\frac{di_\alpha}{dt} &= -\frac{R_s}{L_s} i_\alpha + \frac{\Psi_{PM}}{L_s} \omega_{me} \sin\vartheta + \frac{u_\alpha}{L_s},\\
\frac{di_\beta}{dt} &= -\frac{R_s}{L_s} i_\beta - \frac{\Psi_{PM}}{L_s} \omega_{me} \cos\vartheta + \frac{u_\beta}{L_s},\\
\frac{d\omega}{dt} &= \frac{k_p p_p^2 \Psi_{pm}}{J} \left(i_\beta \cos(\vartheta) - i_\alpha \sin(\vartheta) \right) - \frac{B}{J}\omega - \frac{p_p}{J} T_L,\\
\frac{d\vartheta}{dt} &= \omega_{me}.
\end{aligned}
\tag{1}
$$

Here, i_α, i_β, u_α and u_β represent stator current and voltage in the stationary reference frame, respectively; ω is electrical rotor speed and ϑ is electrical rotor position. R_s and L_s is stator resistance and inductance respectively, Ψ_{pm} is the flux of permanent magnets on the rotor, B is friction and T_L is load torque, J is moment of inertia, p_p is the number of pole pairs, k_p is the Park constant.

The sensor-less control scenario arise when sensors of the speed and position (ω and ϑ) are missing (from various reasons). Then, the only observed variables are:

$$
y_t = \left[\begin{array}{c} i_\alpha(t), i_\beta(t), u_\alpha(t), u_\beta(t) \end{array} \right]. \tag{2}
$$

Which are, however, observed only up to some precision.

Discretization of the model (1) was performed using Euler method with the following result:

$$
\begin{aligned}
i_{\alpha,t+1} &= (1 - \frac{R_s}{L_s}\Delta t) i_{\alpha,t} + \frac{\Psi_{pm}}{L_s} \Delta t \omega_t \sin\vartheta_{e,t} + u_{\alpha,t}\frac{\Delta t}{L_s},\\
i_{\beta,t+1} &= (1 - \frac{R_s}{L_s}\Delta t) i_{\beta,t} - \frac{\Psi_{pm}}{L_s} \Delta t \omega_t \cos\vartheta_t + u_{\beta,t}\frac{\Delta t}{L_s},\\
\omega_{t+1} &= (1 - \frac{B}{J}\Delta t)\omega_t + \Delta t \frac{k_p p_p^2 \Psi_{pm}}{J} \left(i_{\beta,t}\cos(\vartheta_t) - i_{\alpha,t}\sin(\vartheta_t) \right) - \frac{p_p}{J} T_L \Delta t,\\
\vartheta_{t+1} &= \vartheta_t + \omega_t \Delta t.
\end{aligned}
$$

In this work, we consider parameters of the model known, we can make the following substitutions to simplify notation, $a = 1 - \frac{R_s}{L_s}\Delta t$, $b = \frac{\Psi_{pm}}{L_s}\Delta t$, $c = \frac{\Delta t}{L_s}$, $d = 1 - \frac{B}{J}\Delta t$, $e = \Delta t \frac{k_p p_p^2 \Psi_{pm}}{J}$, which results in a simplified model:

$$\begin{aligned} i_{\alpha,t+1} &= a\, i_{\alpha,t} + b\omega_t \sin\vartheta_t + c u_{\alpha,t}, \\ i_{\beta,t+1} &= a\, i_{\beta,t} - b\omega_t \cos\vartheta_t + c u_{\beta,t}, \\ \omega_{t+1} &= d\omega_t + e\left(i_{\beta,t}\cos(\vartheta_t) - i_{\alpha,t}\sin(\vartheta_t)\right), \\ \vartheta_{t+1} &= \vartheta_t + \omega_t \Delta t. \end{aligned} \tag{3}$$

The above equations can be aggregated into state $x_t = [i_{\alpha,t}, i_{\beta,t}, \omega_t, \vartheta_t]$ will be denoted as $x_{t+1} = g(x_t, u_t)$.

1.1 Transformation to d-q coordinates

For many applications, it is advantageous to consider altervative coordinate system denoted d-q as follows

$$\begin{aligned} \begin{bmatrix} d \\ q \end{bmatrix} &= \begin{bmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \\ \begin{bmatrix} \alpha \\ \beta \end{bmatrix} &= \begin{bmatrix} \cos\vartheta & -\sin\vartheta \\ \sin\vartheta & \cos\vartheta \end{bmatrix} \begin{bmatrix} d \\ q \end{bmatrix} \end{aligned}$$

Under this transformation, the whole model (4) can be transformed into d-q coordinates.

In this text, we will transform only one single quantity, L_d and L_q for which it holds $L_d = kL_q$. Then,

$$\begin{aligned} \begin{bmatrix} L_\alpha \\ L_\beta \end{bmatrix} &= \begin{bmatrix} \cos\vartheta & -\sin\vartheta \\ \sin\vartheta & \cos\vartheta \end{bmatrix} \begin{bmatrix} L_d \\ L_q \end{bmatrix}. \\ &= L_d \begin{bmatrix} \cos\vartheta & -\sin\vartheta \\ \sin\vartheta & \cos\vartheta \end{bmatrix} \begin{bmatrix} k \\ 1 \end{bmatrix} \\ &= L \begin{bmatrix} k\cos\vartheta & -\sin\vartheta \\ k\sin\vartheta & \cos\vartheta \end{bmatrix} = L \begin{bmatrix} k_{c\vartheta} \\ k_{s\vartheta} \end{bmatrix}. \end{aligned}$$

Then, model of the drive is changed to

$$\begin{aligned} i_{\alpha,t+1} &= (1 - \frac{R_s}{L_s k_{c\vartheta}}\Delta t) i_{\alpha,t} + \frac{\Psi_{pm}}{L_s k_{c\vartheta}}\Delta t \omega_t \sin\vartheta_{e,t} + u_{\alpha,t}\frac{\Delta t}{L_s k_{c\vartheta}}, \\ i_{\beta,t+1} &= (1 - \frac{R_s}{L_s k_{s\vartheta}}\Delta t) i_{\beta,t} - \frac{\Psi_{pm}}{L_s k_{s\vartheta}}\Delta t \omega_t \cos\vartheta_t + u_{\beta,t}\frac{\Delta t}{L_s k_{s\vartheta}}, \\ \omega_{t+1} &= (1 - \frac{B}{J}\Delta t)\omega_t + \Delta t \frac{k_p p_p^2 \Psi_{pm}}{J}\left(i_{\beta,t}\cos(\vartheta_t) - i_{\alpha,t}\sin(\vartheta_t)\right) - \frac{p_p}{J} T_L \Delta t, \\ \vartheta_{t+1} &= \vartheta_t + \omega_t \Delta t. \end{aligned}$$

Transformation to full d-q

$$
\begin{aligned}
i_{d,t+1} &= (1-\frac{R_s}{L_d}\Delta t)i_{d,t} + \frac{L_q}{L_d}i_{q,t}\Delta t\omega_t + u_{d,t}\frac{\Delta t}{L_d}, \\
i_{q,t+1} &= -\frac{L_d}{L_q}\Delta t\omega_t i_{d,t} + (1-\frac{R_s}{L_q}\Delta t)i_{q,t} - \frac{\Psi_{pm}}{L_q}\Delta t\omega_t + u_{q,t}\frac{\Delta t}{L_q}, \\
\omega_{t+1} &= \underbrace{(1-\frac{B}{J}\Delta t)}_{\approx 1}\omega_t + \Delta t\frac{k_p p_p^2}{J}((L_d - L_q)i_d + \Psi_{pm})i_q \\
\vartheta_{t+1} &= \vartheta_t + \Delta t\omega_t
\end{aligned}
$$

Observation:

$$
\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} \cos\vartheta & -\sin\vartheta \\ \sin\vartheta & \cos\vartheta \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + e_t
$$

1.2 Gaussian model of disturbances

This model is motivated by the well known Kalman filter, which is optimal for linear system with Gaussian noise. Hence, we model all disturbances to have covariance matrices Q_t and R_t for the state x_t and observations y_t respectively.

$$
\begin{aligned}
x_{t+1} &\sim \mathcal{N}(g(x_t), Q_t) \\
y_t &\sim \mathcal{N}([i_{\alpha,t}, i_{\beta,t}]', R_t)
\end{aligned}
\tag{4}
$$

Under this assumptions, Bayesian estimation of the state, x_t, can be approximated by so called Extended Kalman filter which approximates posterior density of the state by a Gaussian

$$
f(x_t|y_1 \ldots y_t) = \mathcal{N}(\hat{x}_t, S_t).
$$

Its sufficient statistics $S_t = [\hat{x}_t, P_t]$ is evaluated recursively as follows:

$$
\begin{aligned}
\hat{x}_t &= g(\hat{x}_{t-1}) - K\left(y_t - h(\hat{x}_{t-1})\right). && (5) \\
R_y &= CP_{t-1}C' + R_t, \\
K &= P_{t-1}C'R_y^{-1}, \\
S_t &= P_{t-1} - P_{t-1}C'R_y^{-1}CP_{t-1}, && (6) \\
P_t &= AS_tA' + Q_t. && (7)
\end{aligned}
$$

where $A = \frac{d}{dx_t}g(x_t)$, $C = \frac{d}{dx_t}h(x_t)$, $g(x_t)$ is model (3) and $h(x_t)$ direct observation of $y_t = [i_{\alpha,t}, i_{\beta,t}]$, i.e.

$$
A = \begin{bmatrix} a & 0 & b\sin\vartheta & b\omega\cos\vartheta \\ 0 & a & -b\cos\vartheta & b\omega\sin\vartheta \\ -e\sin\vartheta & e\cos\vartheta & d & -e(i_\beta\sin\vartheta + i_\alpha\cos\vartheta) \\ 0 & 0 & \Delta t & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}
$$

$$B = \begin{bmatrix} c & 0 \\ 0 & c \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Covariance matrices of the system Q and R are supposed to be known.

1.2.1 Reduced order version

Equations (4) ca be restructured by considering $i_{s\alpha}$ and $i_{s\beta}$ as external observations. Then the state variables are $x_t = [\omega_t, \vartheta_t]$ and as follows:

$$\begin{aligned} \omega_{t+1} &= d\omega_t + e\left(i_{\beta,t}\cos(\vartheta_t) - i_{\alpha,t}\sin(\vartheta_t)\right), \\ \vartheta_{t+1} &= \vartheta_t + \omega_t \Delta t. \end{aligned} \tag{8}$$

and the onbservation equations are

$$\begin{aligned} i_{\alpha,t+1} &= a\, i_{\alpha,t} + b\omega_t \sin\vartheta_t + cu_{\alpha,t}, \\ i_{\beta,t+1} &= a\, i_{\beta,t} - b\omega_t \cos\vartheta_t + cu_{\beta,t}, \end{aligned} \tag{9}$$

These equations are used by the EKF to update estimates of mean values. The new matrices A and C are

$$A = \begin{bmatrix} d & -e(i_\beta \sin\vartheta + i_\alpha \cos\vartheta) \\ \Delta t & 1 \end{bmatrix}, \quad C = \begin{bmatrix} b\sin\vartheta & b\omega\cos\vartheta \\ -b\cos\vartheta & b\omega\sin\vartheta \end{bmatrix}.$$

1.3 Test system

A real PMSM system on which the algorithms will be tested has parameters:

$$\begin{aligned} R_s &= 0.28; \\ L_s &= 0.003465; \\ \Psi_{pm} &= 0.1989; \\ k_p &= 1.5 \\ p &= 4.0; \\ J &= 0.04; \\ \Delta t &= 0.000125 \end{aligned}$$

which yields

$$\begin{aligned} a &= 0.9898 \\ b &= 0.0072 \\ c &= 0.0361 \\ d &= 1 \\ e &= 0.0149 \end{aligned}$$

The covaraince matrices Q and R are assumed to be known. For the initial tests, we can use the following values:

$$\begin{aligned} Q &= \mathrm{diag}(0.0013, 0.0013, 5e-6, 1e-10), \\ R &= \mathrm{diag}(0.0006, 0.0006). \end{aligned}$$

Limits:

$$\begin{aligned} u_{\alpha,max} &= 50V, & u_{\alpha,min} &= -50V, \\ u_{\beta,max} &= 50V. & u_{\beta,min} &= -50V, \end{aligned}$$

Perhaps better:

$$u_\alpha^2 + u_\beta^2 < 100^2.$$

2 Control

The task is to reach predefined speed $\overline{\omega}_t$.

For simplicity, we will assume additive loss function:

$$\begin{aligned} l(x_t, u_t) &= (\omega_t - \overline{\omega}_t)^2 + q(u_{\alpha,t}^2 + u_{\beta,t}^2). \\ &= (\omega_t - \overline{\omega}_t)\Xi(\omega_t - \overline{\omega}_t) + [u_{\alpha t}, u_{\beta t}] \underbrace{\begin{bmatrix} \upsilon & 0 \\ 0 & \upsilon \end{bmatrix}}_{\Upsilon} \begin{bmatrix} u_{\alpha t} \\ u_{\beta t} \end{bmatrix} \end{aligned}$$

Here, Υ is the chosen penalization of the inputs, which remains to be tuned.

Note: classical notation of penalization matrices is Q and R, but it conflicts wit Q and R in (4).

Following the standard dynamic programming approach, optimization of the loss function can be done recursively, as follows:

$$V(x_{t-1}, u_{t-1}) = \arg\min_{u_t} \mathsf{E}_{f(x_t, y_t|x_{t-1})} \left\{ l(x_t, u_t) + V(x_t, u_t) \right\},$$

where $V(x_t, u_t)$ is the Bellman function. Since the model evolution is stochastic, we can reformulate it in terms of sufficient statistics, S as follows:

$$V(S_{t-1}) = \min_{u_t} \mathsf{E}_{f(x_t, y_t|x_{t-1})} \left\{ l(x_t, u_t) + V(S_t) \right\}.$$

Representation of the Bellman function depends on chosen approximation.

2.1 LQG control

Control of linear state-space model with Gaussian noise

$$\begin{aligned} x_t &= Ax_{t-1} + Bu_t + Q^{\frac{1}{2}} v_t, \\ y_t &= Cx_t + Du_t + R^{\frac{1}{2}} w_t. \end{aligned}$$

to minimize loss function

$$L_t = (x_t - \overline{x}_t)' \Xi (x_t - \overline{x}_t) + u_t' \Upsilon u_t.$$

Optimal solution in the sense of dynamic programming on horizon $t + h$ is:

$$\begin{aligned} u_t &= L_t \left(\hat{x}_t - \overline{x}_t \right), \\ L_t &= -(B' S_{t+1} B + \Upsilon)^{-1} B' S_{t+1} A, \\ S_t &= A'(S_{t+1} - S_{t+1} B (B' S_{t+1} B + \Upsilon)^{-1} B' S_{t+1}) A + \Xi, \end{aligned}$$

This solution is certainty equivalent, i.e. only the first moment, $\hat{x}$, of the Kalman filter is used.

2.2 PI control

The classical control is based on transformation to dq reference frame:

$$\begin{aligned} i_d &= i_\alpha \cos(\vartheta) + i_\beta \sin(\vartheta), \\ i_q &= i_\beta \cos(\vartheta) - i_\alpha \sin(\vartheta). \end{aligned}$$

Desired i_q current, $\bar{i}_q$, is derived using PI controller

$$\bar{i}_q = PI(\overline{\omega} - \omega, P_i, I_i).$$

This current needs to be achieved through voltages u_d, u_q which are again obtained from a PI controller

$$\begin{aligned} u_d &= PI(-i_d, P_u, I_u), \\ u_q &= PI(\bar{i}_q - i_q, P_u, I_u). \end{aligned}$$

These are compensated (for some reason) as follows:

$$\begin{aligned} u_d &= u_d - L_S \omega \bar{i}_q, \\ u_q &= u_q + \Psi_{pm} \omega. \end{aligned}$$

Conversion to u_α, u_β is

$$u_\alpha = |U| \cos(\phi), \qquad u_\beta = |U| \sin(\phi)$$

$$|U| = \sqrt{u_d^2 + u_q^2}, \qquad \phi = \begin{cases} \arctan(\frac{u_q}{u_d}) + \vartheta & u_d \geq 0 \\ \arctan(\frac{u_q}{u_d}) + \pi + \vartheta & u_d < 0 \end{cases}$$

PI controller is defined as follows:

$$\begin{aligned} x &= PI(\epsilon, P, I) \\ &= P\epsilon + I(S_{t-1} + \epsilon) \\ S_t &= S_{t-1} + \epsilon \end{aligned}$$

Constants for the system:

$$P_i = 3,\ I_i = 0.00375,\ P_u = 20,\ I_u = 0.5.$$

The requested values for ω should be kept in interval $< -30, 30 >$.

2.3 Cautious LQG control

Uncertainty in A.

Sigma points: $x^{(i)} = \hat{x} + hv_i$, v_i are eigenvectors of P.

$$\begin{aligned} E\{x'A(x)QA(x)x\} &= \frac{1}{n}\sum x'A(x^{(i)})QA^{(i)}(x)x \\ &= x'Zx' \end{aligned}$$

Uncented transform...

2.4 Poor-man's dual LQG control

Various heuristic solutions to dual extension of LQG has been proposed. Most of them is based on approximation of the loss function

$$L_t = (x_t - \overline{x}_t)'\Xi(x_t - \overline{x}_t) + (u_t - \overline{u}_t)'\Upsilon(u_t - \overline{u}_t) + DUAL\_TERM.$$

where DUAL_TERM is typically a function of P_{t+2}.

To be continued...

2.5 Test Scenarios

With almost full information, design of the control strategy should be almost trivial:

$$\begin{aligned} \hat{i_\alpha} &= 0, \hat{i_\beta} = 0, \hat{\omega} = 1, \vartheta = \frac{\pi}{2}, \\ P_t &= \text{diag}([0.01, 0.01, 0.01, 0.01]). \end{aligned}$$

The difficulty arise with growing initial covariance matrix:

$$\begin{aligned} \hat{i_\alpha} &= 0, \hat{i_\beta} = 0, \hat{\omega} = 1, \vartheta = \frac{\pi}{2}, \\ P_t &= \text{diag}([0.01, 0.01, 0.01, 1]). \end{aligned}$$

Or even worse:

$$\begin{aligned}
\hat{i_\alpha} &= 0, \hat{i_\beta} = 0, \hat{\omega} = 1, \vartheta = \frac{\pi}{2}, \\
P_t &= \text{diag}([0.01, 0.01, 0.01, 10]).
\end{aligned}$$

===

The requested value $\overline{\omega}_t = 1.0015$.

Conjecture 1. *It is sufficient to consider hyper-state* $H = [\hat{i}_\alpha, \hat{i}_\beta, \hat{\omega}, \hat{\vartheta}, P(3,3), P(4,4)]$.

References