System pro simulaci

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Puvodni zadani:

Vychazime ze zadani [2]:

$$\begin{aligned} SYSTEM: \; y_{t+1} &= y_t + bu_t + \sigma e_t, \;\; e_t \sim \mathcal{N}(0,1), \\ ZTRATA: \;\; L_t &= (y_{t+1} - r_{t+1})^2 \end{aligned} \tag{1}$$

Reseni schematicky:

$$V_t = \min_{u_t} \mathsf{E}_{e_t,b} \left\{ L_t + V_{t+1} | y_t, u_{t-1}, y_{t-1}, u_{t-2}, \ldots \right\} \tag{2}$$

kde stredni hodnota se pocita pres neurcitost v e_t a pres neurcitost v b.

Pro linearni a Gausovsky system (1) je k dispozici konjugovana hustota ve forme Normalniho rozlozeni pravdepodobnosti $f(b_t) = \mathcal{N}(\hat{b}_t, P_t)$, jejiz parametry se vyvijeji rekurzivne, rovnice (28) v [2]. Tim padem je mozne vycislit ocekavanou hodnotu pres b v (2) analyticky:

$$\begin{aligned} V_t &= \min_{u_t} \mathsf{E}_{e_t,b} \left\{ (y_t + bu_t + \sigma e_t - r_{t+1})^2 + V_{t+1} | y_t, u_{t-1}, y_{t-1}, u_{t-2}, \ldots \right\} \\ &= \min_{u_t} \mathsf{E}_{e_t} \left\{ (y_t + \hat{b} u_t + \sigma e_t - r_{t+1})^2 + P_t u_t^2 | y_t, u_{t-1}, y_{t-1}, u_{t-2}, \ldots \right\} + \\ &\quad + \mathsf{E}_{e_t,b} \left\{ V_{t+1} | y_t, u_{t-1}, y_{t-1}, u_{t-2}, \ldots \right\} \end{aligned}$$

Muzeme provest preznaceni

$$V_{t+1}(H_t) = \mathsf{E}_{e_t,b} \left\{ V_{t+1} | y_t, u_{t-1}, y_{t-1}, u_{t-2}, \ldots \right\}$$

kde $H_t = [y_t, \hat{b}_t, P_t]$. Vysledna uloha je ekvivalentni tomu, kdyby zadani bylo:

$$\begin{aligned} SYSTEM: \; H_{t+1} &= \begin{bmatrix} y_{t+1} \\ \hat{b}_{t+1} \\ P_{t+1} \end{bmatrix} = \begin{bmatrix} y_t + \hat{b}_t u_t \\ (28) \\ (28) \end{bmatrix} + \begin{bmatrix} \sigma e_t \\ 0 \\ 0 \end{bmatrix} \\ ZTRATA: \;\; L_t &= (y_{t+1} - r_{t+1})^2 + P_t u_t^2. \end{aligned}$$

(28) je opet rovnice (28) z [2]. Takto upravenou ulohu muzeme resit pomoci algoritmu [3].[1]

References

[1] D.P. Bertsekas. *Dynamic Programming and Optimal Control*. Athena Scientific, Nashua, US, 2001. 2nd edition.

[2] A.M. Thompson and W.R. Cluet. Stochastic iterative dynamic programming: a monte carlo approach to dual control. *Automatica*, 41:767–778, 2005.

[3] E. Todorov and Tassa. Y. Iterative local dynamic programming. In *Proceedings of the 2nd IEEE Symposium on Adaptive Dynamic Programming and Reinforcement Learning*, pages 90 – 95, 2009.