PMSM system description

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1 Model of PMSM Drive

Permanent magnet synchronous machine (PMSM) drive with surface magnets on the rotor is described by conventional equations of PMSM in the stationary reference frame:

$$\begin{aligned}
\frac{di_\alpha}{dt} &= -\frac{R_s}{L_s} i_\alpha + \frac{\Psi_{PM}}{L_s}\omega_{me}\sin\vartheta + \frac{u_\alpha}{L_s},\\
\frac{di_\beta}{dt} &= -\frac{R_s}{L_s} i_\beta - \frac{\Psi_{PM}}{L_s}\omega_{me}\cos\vartheta + \frac{u_\beta}{L_s},\\
\frac{d\omega}{dt} &= \frac{k_p p_p^2 \Psi_{pm}}{J}\left(i_\beta\cos(\vartheta) - i_\alpha\sin(\vartheta)\right) - \frac{B}{J}\omega - \frac{p_p}{J}T_L,\\
\frac{d\vartheta}{dt} &= \omega_{me}.
\end{aligned} \tag{1}$$

Here, i_α, i_β, u_α and u_β represent stator current and voltage in the stationary reference frame, respectively; ω is electrical rotor speed and ϑ is electrical rotor position. R_s and L_s is stator resistance and inductance respectively, Ψ_{pm} is the flux of permanent magnets on the rotor, B is friction and T_L is load torque, J is moment of inertia, p_p is the number of pole pairs, k_p is the Park constant.

The sensor-less control scenario arise when sensors of the speed and position (ω and ϑ) are missing (from various reasons). Then, the only observed variables are:

$$y_t = \left[\begin{array}{c} i_\alpha(t), i_\beta(t), u_\alpha(t), u_\beta(t) \end{array}\right]. \tag{2}$$

Which are, however, observed only up to some precision.

Discretization of the model (1) was performed using Euler method with the following result:

$$\begin{aligned}
i_{\alpha,t+1} &= (1 - \frac{R_s}{L_s}\Delta t) i_{\alpha,t} + \frac{\Psi_{pm}}{L_s}\Delta t \omega_t \sin\vartheta_{e,t} + u_{\alpha,t}\frac{\Delta t}{L_s},\\
i_{\beta,t+1} &= (1 - \frac{R_s}{L_s}\Delta t) i_{\beta,t} - \frac{\Psi_{pm}}{L_s}\Delta t \omega_t \cos\vartheta_t + u_{\beta,t}\frac{\Delta t}{L_s},\\
\omega_{t+1} &= (1 - -\frac{B}{J}\Delta t)\omega_t + \Delta t \frac{k_p p_p^2 \Psi_{pm}}{J}\left(i_{\beta,t}\cos(\vartheta_t) - i_{\alpha,t}\sin(\vartheta_t)\right) - \frac{p_p}{J}T_L\Delta t,\\
\vartheta_{t+1} &= \vartheta_t + \omega_t \Delta t.
\end{aligned}$$

In this work, we consider parameters of the model known, we can make the following substitutions to simplify notation, $a = 1 - \frac{R_s}{L_s}\Delta t$, $b = \frac{\Psi_{pm}}{L_s}\Delta t$, $c = \frac{\Delta t}{L_s}$, $d = 1 - \frac{B}{J}\Delta t$, $e = \Delta t \frac{k_p p_p^2 \Psi_{pm}}{J}$, which results in a simplified model:

$$\begin{aligned} i_{\alpha,t+1} &= a\, i_{\alpha,t} + b\omega_t \sin\vartheta_t + c u_{\alpha,t}, \\ i_{\beta,t+1} &= a\, i_{\beta,t} - b\omega_t \cos\vartheta_t + c u_{\beta,t}, \\ \omega_{t+1} &= d\omega_t + e\left(i_{\beta,t}\cos(\vartheta_t) - i_{\alpha,t}\sin(\vartheta_t)\right), \\ \vartheta_{t+1} &= \vartheta_t + \omega_t \Delta t. \end{aligned} \tag{3}$$

The above equations can be aggregated into state $x_t = [i_{\alpha,t}, i_{\beta,t}, \omega_t, \vartheta_t]$ will be denoted as $x_{t+1} = g(x_t, u_t)$.

1.1 Gaussian model of disturbances

This model is motivated by the well known Kalman filter, which is optimal for linear system with Gaussian noise. Hence, we model all disturbances to have covariance matrices Q_t and R_t for the state x_t and observations y_t respectively.

$$\begin{aligned} x_{t+1} &\sim \mathcal{N}(g(x_t), Q_t) \\ y_t &\sim \mathcal{N}([i_{\alpha,t}, i_{\beta,t}]', R_t) \end{aligned}$$

Under this assumptions, Bayesian estimation of the state, x_t, can be approximated by so called Extended Kalman filter which approximates posterior density of the state by a Gaussian

$$f(x_t|y_1 \ldots y_t) = \mathcal{N}(\hat{x}_t, P_t).$$

Its sufficient statistics $S_t = [\hat{x}_t, P_t]$ is evaluated recursively as follows:

$$\hat{x}_t = g(\hat{x}_{t-1}) - K\left(y_t - h(\hat{x}_{t-1})\right). \tag{4}$$

$$\begin{aligned} R_y &= C' P_{t-1} C + R_t, \\ K &= P_{t-1} C R_y^{-1} (y_t - h(\hat{x}_{t-1})), \end{aligned}$$

$$P_t = A\left(P_{t-1} - P_{t-1} C' R_y^{-1} C P_{t-1}\right) A + Q_t. \tag{5}$$

where $A = \frac{d}{dx_t} g(x_t)$, $C = \frac{d}{dx_t} h(x_t)$, $g(x_t)$ is model (3) and $h(x_t)$ direct observation of $y_t = [i_{\alpha,t}, i_{\beta,t}]$, i.e.

$$A = \left[\begin{array}{cccc} a & 0 & b\sin\vartheta & b\omega\cos\vartheta \\ 0 & a & -b\cos\vartheta & b\omega\sin\vartheta \\ -e\sin\vartheta & e\cos\vartheta & d & -e(i_\beta \sin\vartheta + i_\alpha \cos\vartheta) \\ 0 & 0 & \Delta t & 1 \end{array}\right], \quad C = \left[\begin{array}{cccc} 1 & & & \\ & 1 & & \\ & & 0 & \\ & & & 0 \end{array}\right]$$

Covariance matrices of the system Q and R are supposed to be known.

1.2 Test system

A real PMSM system on which the algorithms will be tested has parameters:

$$\begin{aligned} R_s &= 0.28; \\ L_s &= 0.003465; \\ \Psi_{pm} &= 0.1989; \\ k_p &= 1.5 \\ p &= 4.0; \\ J &= 0.04; \\ \Delta t &= 0.000125 \end{aligned}$$

which yields

$$\begin{aligned} a &= 0.9898 \\ b &= 0.0072 \\ c &= 0.0361 \\ d &= 1 \\ e &= 0.0149 \end{aligned}$$

The covaraince matrices Q and R are assumed to be known. For the initial tests, we can use the following values:

$$\begin{aligned} Q &= \operatorname{diag}(0.0013, 0.0013, 5e-6, 1e-10), \\ R &= \operatorname{diag}(0.0006, 0.0006). \end{aligned}$$

2 Control

The task is to reach predefined speed $\overline{\omega}_t$.

For simplicity, we will assume additive loss function:

$$l(x_t, u_t) = (\omega_t - \overline{\omega}_t)^2 + \psi(u_{\alpha,t}^2 + u_{\beta,t}^2).$$

Here, ψ is the chosen penalization of the inputs.

Following the standard dynamic programming approach, optimization of the loss function can be done recursively, as follows:

$$V(x_{t-1}, u_{t-1}) = \arg\min_{u_t} \mathsf{E}_{f(x_t, y_t|x_{t-1})} \left\{ l(x_t, u_t) + V(x_t, u_t) \right\},$$

where $V(x_t, u_t)$ is the Bellman function. Since the model evolution is stochastic, we can reformulate it in terms of sufficient statistics, S as follows:

$$V(S_{t-1}) = \min_{u_t} \mathsf{E}_{f(x_t, y_t|x_{t-1})} \left\{ l(x_t, u_t) + V(S_t) \right\}.$$

Representation of the Bellman function depends on chosen approximation.

2.1 Test Scenarios

With almost full information, design of the control strategy should be almost trivial:

$$\begin{aligned}\hat{i_\alpha} &= 0,\ \hat{i_\beta} = 0, \hat{\omega} = 1, \vartheta = \frac{\pi}{2},\\ P_t &= \mathrm{diag}([0.01, 0.01, 0.01, 0.01]).\end{aligned}$$

The difficulty arise with growing initial covariance matrix:

$$\begin{aligned}\hat{i_\alpha} &= 0,\ \hat{i_\beta} = 0, \hat{\omega} = 1, \vartheta = \frac{\pi}{2},\\ P_t &= \mathrm{diag}([0.01, 0.01, 1, 1]).\end{aligned}$$

Or even worse:

$$\begin{aligned}\hat{i_\alpha} &= 0,\ \hat{i_\beta} = 0, \hat{\omega} = 1, \vartheta = \frac{\pi}{2},\\ P_t &= \mathrm{diag}([0.01, 0.01, 1, 10]).\end{aligned}$$

The requested value $\overline{\omega}_t = 3$.

References